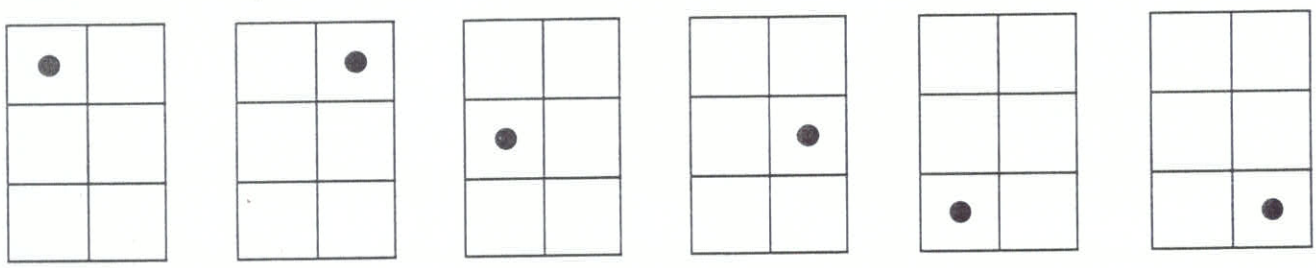
9 Braille

What you need: Squared paper marked into 2 × 3 rectangles, pencil

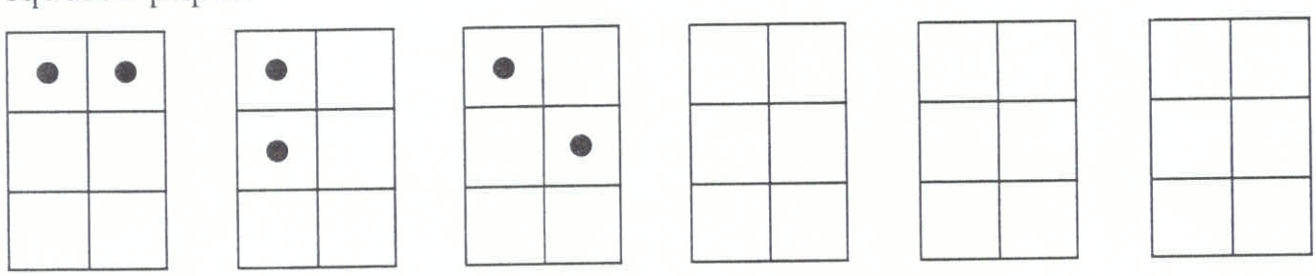
When Louis Braille invented his method of reading by touch, he needed a way of arranging a number of dots in a particular array. He chose to use a 3 × 2 rectangular array. In this he could distribute one, two, three, four, five or six raised dots.

He had to find out how many ways he could put one dot in an array.

There were six, like this:



How many ways could he put two dots?
Use these rectangles to investigate, then continue on squared paper.



Investigate the ways he could have arranged three, four, five or six dots. How about 0 dots?

How many different ways are there altogether of arranging dots in a 3 × 2 array?

If each arrangement of dots stood for a letter of the alphabet, a punctuation mark or a number, could he have managed with a 2 × 2 array or a 5 × 1 array?

How many different arrangements are possible with a 7 × 1 array?

Braille

Equipment Resource sheet 1, pencil

Facts that will aid investigation Term: array

Skills Working methodically, noting patterns

Concepts Symmetry in number, number patterns

Teaching points

This investigation is as much about working methodically as it is about number patterns. Finding the number of ways of arranging two dots without repeating some arrangements or missing some out can be difficult unless some sort of logical approach, like that hinted at in the diagrams on the pupils' page, is adopted. However, it is important that children should initially work in their own ways – comparing results with other children, together with discussion in groups, should enable the children to sort out a strategy.

Further assistance can be given by asking,

'Will it help if we first look at all the ways of arranging two dots when one is always in the top left-hand corner?' There are in fact, five. There are four ways when we move the fixed point to another square, then three, then two, then one, giving a total of 15. Can the children explain why this is so?

The results for four dots will be found to be the same as for two dots. Likewise results for five and one dots are the same, and for 0 and six dots. This symmetry should be discussed. The reason for it is that arranging two dots is the same as arranging two spaces – that is, four dots.

The results are as follows:

0 dots		1 possible arrangement
1		6
2		15
3		20
4		15
5		6
6		1
	Total	64

The children can now work out the results for arrays of five, four, three, two and one square(s) which they can usually do quite quickly once they have discovered a methodical way of working. When the results are arranged like this:

		Total number of ways
1 square	1 1	2
2 squares	1 2 1	4
3 squares	1 3 3 1	8
4 squares	1 4 6 4 1	16
5 squares	1 5 10 10 5 1	32
6 squares	1 6 15 20 15 6 1	64

we obtain a very familiar arrangement – Pascal's triangle again! (See 'Routes', Book 2).

The number of arrangements for a seven-square array is 128, but the children won't have to work out the whole of the seventh row to work that out! The totals for each row can be seen to double each time.

Extension activities

A good way to follow this up is to tackle 'Looking forward' (Investigation 16). Children could also find out about Louis Braille and his system. Are all 64 possible arrangements actually used in the Braille system?

Related investigations 'Looking forward' (Investigation 16), 'Three in a line' (Book 2), 'Eggs in a box' (Book 3)